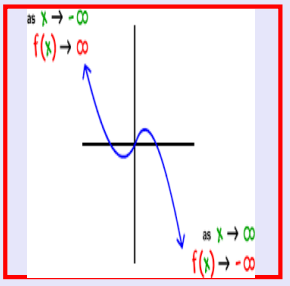
Math 245

Spring 2022

Lecture 27



Class QZ 9

Solve $2x^2 + 5 = -2x$ by using the quadratic formula.

$ax^2 + bx + c = 0$

$2x^2 + 5 = -2x \Rightarrow 2x^2 + 5 + 2x = 0 \Rightarrow \boxed{2x^2 + 2x + 5 = 0}$

$a = 2, b = 2, c = 5$

$b^2 - 4ac = 2^2 - 4(2)(5) = -36$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{-36}}{2(2)} = \frac{-2 \pm 6i}{4} = \frac{-2}{4} \pm \frac{6}{4}i$$

$$x = -\frac{1}{2} \pm \frac{3}{2}i \Rightarrow \boxed{\left\{-\frac{1}{2} \pm \frac{3}{2}i\right\}}$$

Discriminant b^2-4ac

b^2-4ac
- >0 Two Real Solutions
- $=0$ One repeated real solution
- <0 two comple number Solutions

Ex: Determine the type of solutions for

$(3x+2)(2x-5)=10.$

Foil, Simplify, write in quadratic form

$6x^2-15x+4x-10=10$

$6x^2-11x-20=0$

$a=6 \quad b=-11 \quad c=-20$

$b^2-4ac=(-11)^2-4(6)(-20)=121+480=601$

Since $b^2-4ac>0 \Rightarrow$ Two Real Solutions

Determine the type of Solutions

$4x^2+26=20x$

$4x^2-20x+26=0$

Divide by 2 to reduce

$2x^2-10x+13=0$

$a=2 \quad b=-10 \quad c=13$

$b^2-4ac=(-10)^2-4(2)(13)=100-104=-4$

Since $b^2-4ac<0 \Rightarrow$ Two complex number Solutions.

Find the value of the discriminant of $9x^2 = 6x - 1$, discuss its solutions, then solve.

$9x^2 - 6x + 1 = 0$ $\quad a=9,\ b=-6,\ c=1$

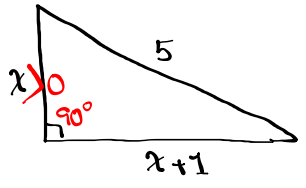
$b^2 - 4ac = (-6)^2 - 4(9)(1) = 36 - 36 = \boxed{0}$

Since $b^2 - 4ac = 0 \Rightarrow$ one repeated real solution

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-6) \pm \sqrt{0}}{2(9)} = \frac{6 \pm 0}{18} = \frac{6}{18} = \boxed{\frac{1}{3}}$$

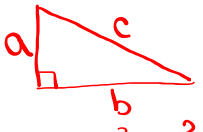
Solution Set $\left\{\frac{1}{3}\right\}$

Find x:



Right Triangle $\Rightarrow$ one angle $= 90°$

Pythagorean theorem



$a^2 + b^2 = c^2$

$x^2 + (x+1)^2 = 5^2$

$x^2 + (x+1)(x+1) = 25$

$x^2 + x^2 + x + x + 1 - 25 = 0$

$\Rightarrow 2x^2 + 2x - 24 = 0$

$x^2 + x - 12 = 0$

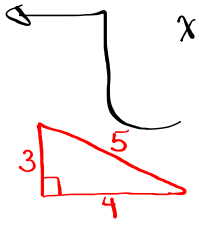
$a=1 \quad b=1 \quad c=-12$

$b^2 - 4ac = 1^2 - 4(1)(-12) = 49$

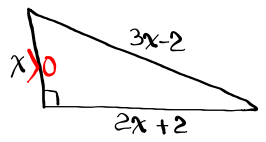
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{49}}{2(1)} = \frac{-1 \pm 7}{2}$$

$x = \frac{-1+7}{2} = \frac{6}{2} = 3$

$x = \frac{-1-7}{2} = \frac{-8}{2} = -4$ ✗



Find all three sides of the triangle below:



Right Triangle

Pythagorean Thm

$x^2 + (2x+2)^2 = (3x-2)^2$

$x^2 + 4x^2 + 8x + 4 = 9x^2 - 12x + 4$

$5x^2 + 8x = 9x^2 - 12x \Rightarrow 9x^2 - 12x - 5x^2 - 8x = 0$

$4x^2 - 20x = 0$

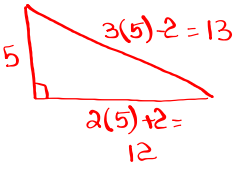
Divide by 4

$x^2 - 5x = 0$

Factor $x(x-5) = 0$

$x = 0$ or $x - 5 = 0$

$x = 5$



Three sides are 5, 12, and 13.

Check: $5^2 + 12^2 = 13^2$

$25 + 144 = 169$

$169 = 169$

SG 10 ✓

Solve $x^2 + 8x - 12 = 0$ by the completing the square method.

$x^2 + 8x + 4^2 = 12 + 4^2$

$\frac{1}{2} \cdot 8 = 4$

$x^2 + 8x + 16 = 28$

$(x+4)^2 = 28$

Use S.R.M.

$x + 4 = \pm\sqrt{28}$

$x = -4 \pm \sqrt{4}\sqrt{7}$

$\boxed{x = -4 \pm 2\sqrt{7}}$

$\{-4 \pm 2\sqrt{7}\}$

Solve by completing the square method:

$3x^2 - 2x - 5 = 0$

$3x^2 - 2x = 5$

Divide by 3

$\frac{3}{3}x^2 - \frac{2}{3}x = \frac{5}{3}$

$x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2 = \frac{5}{3} + \left(\frac{1}{3}\right)^2$

$\frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$

$x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{5}{3} + \frac{1}{9}$

$\left(x - \frac{1}{3}\right)^2 = \frac{16}{9}$

Now S.R.M.

$x - \frac{1}{3} = \pm\sqrt{\frac{16}{9}}$

$x = \frac{1}{3} \pm \frac{4}{3}$

$x = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$

$x = \frac{1}{3} - \frac{4}{3} = -\frac{3}{3} = -1$

$\left\{-1, \frac{5}{3}\right\}$

Solve $(5x - 2)^2 = -20$

Using S.R.M.

$5x - 2 = \pm\sqrt{-20}$

$5x = 2 \pm \sqrt{4}\sqrt{5}\sqrt{-1}$

$5x = 2 \pm 2\sqrt{5}\,i$

$x = \frac{2}{5} \pm \frac{2\sqrt{5}}{5}i$

Solution Set $\left\{\frac{2}{5} \pm \frac{2\sqrt{5}}{5}i\right\}$

Real Part

Imaginary Part

No class on Thursday

Spring Break is Next week.